

# Chapter 26

## Models for Precession Electron Diffraction

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**Abstract** Precession Electron Diffraction has become an increasingly popular method of obtaining crystallographic data, and may well replace older methods such as selected area diffraction or microdiffraction. While a full model has to involve a dynamical calculation, some approximations give some indication how the results vary as a function of thickness and precession angle. This note reviews some of the basic models, their advantages and failures as well as some of the open issues.

### 26.1 Introduction

Over the last few years Precession Electron Diffraction (PED), a technique for acquiring electron diffraction intensities, invented in 1994 by Vincent and Midgley [1] has started to emerge as a viable technique for determining structures based solely upon the intensities, and/or with some assistance from crystallographic phases determined using HREM or similar techniques. An incomplete list of references is [1–50]. It was clear from the first attempts to use the method coupled with direct methods that it gave remarkably better results than conventional diffraction techniques except in relatively special cases such as surfaces where the diffraction intensities are very close to kinematical. Hence the quandary; electron diffraction can only be properly be described using dynamical diffraction, but tools based upon a kinematical formulation work. Why? While the detailed answer to this is still not fully understood, many of the details are and I will here briefly describe the main models along with their advantages and limitations.

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## 26.2 Kinematical Model

The kinematical model has to be mentioned as it is the simplest. The result one gets is that the intensities are proportional to the square of the crystallographic structure factor. Unfortunately except for special cases such as surfaces or graphene monolayers the method has only a very limited relevance for standard samples as an accurate model, as illustrated in Fig. 26.1, failing by 10 nm thickness.

## 26.3 Blackman Model

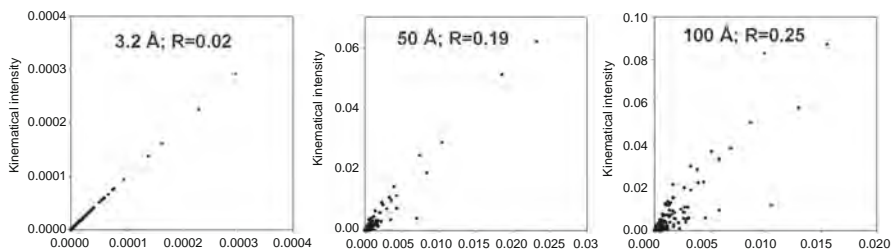
The Blackman model [51, 52] makes the assumption that the integration over angles can be considered as equivalent to a complete integration of a two-beam diffraction problem for all possible angles. In more detail, the intensity for a given reflection can be written as the integral of a Bessel function:

$$I(t) = \int_0^{A_g} J_0(2x) dx \quad A_g = \frac{\pi t}{\xi_g^2}$$

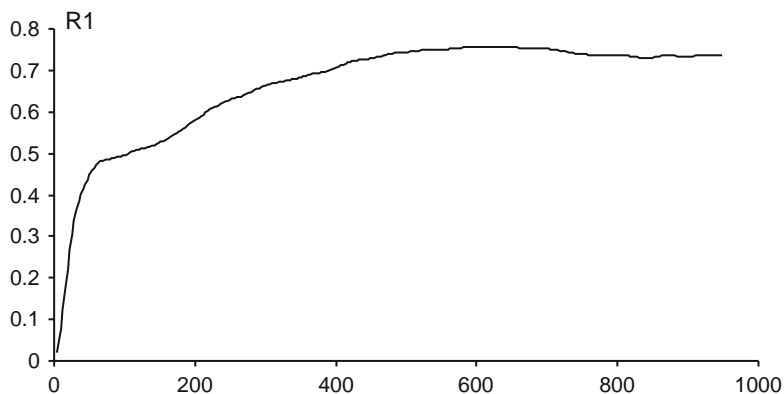
Where  $\xi_g$  is the standard extinction distance which scales inversely with the structure factor and  $t$  is the thickness. The result one obtains is that the intensity, for a relatively thick crystal, scales directly as the crystallographic structure factor; for a thin crystal it scales as the square of the structure factors. While this is again a useful, simple approximation which has been sometimes used and is better than kinematical, there are several fundamental problems with it:

- It neglects most dynamical diffraction effects, as the two-beam model really only applies for specific orientations.
- It neglects the fact that in a precession experiment only a limited range of angles are used.

Unfortunately it is not accurate,  $R1 > 40\%$  for  $200 \text{ \AA}$ , see Fig. 26.2.



**Fig. 26.1** Comparison of kinematical intensities (y axis) versus full dynamical calculations (x axis) for different thicknesses for  $(\text{Ga,In})_2\text{SnO}_4$  with the  $R1$  shown



**Fig. 26.2** Values of the R1 from a Blackman model versus a full dynamical calculation (y-axis) for  $(\text{Ga,In})_2\text{SnO}_4$  as a function of thickness in Angstroms along the x-axis

## 26.4 Methods Based Upon Lorentz-Type Corrections

From the earliest days of PED a different approach has been to try and separate the contributions associated with the integration over angle and dynamical diffraction effects, what has been called a Lorentz correction. In more formal fashion, the intensity would be written as

$$I(g) = L(g) * B(g)$$

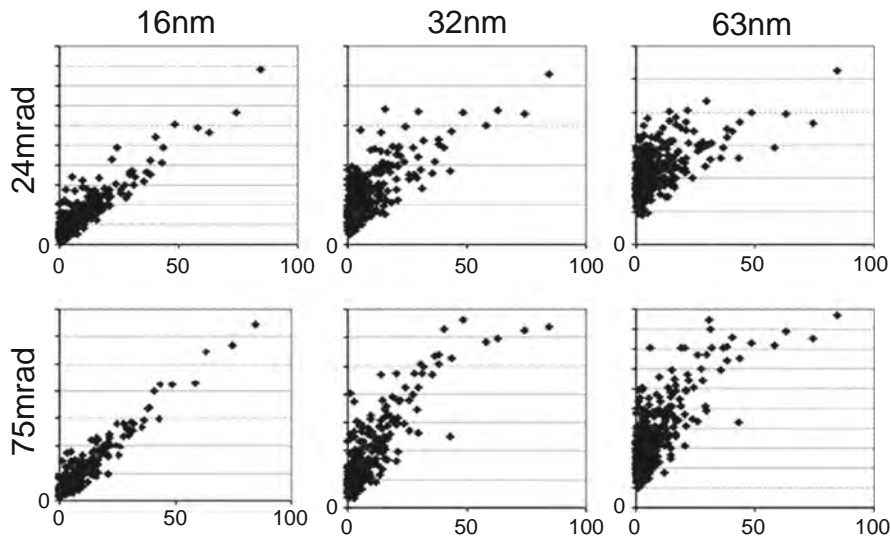
where  $L(g)$  is an approximate form to take into account the integration range, and  $B(g)$  is purely a diffraction term, for instance Kinematical or the Blackman equation. The concept is that one might then be able to precalculate  $L(g)$  and remove it, thereby obtaining a better form. A simple form for  $L(g)$  suggested by Gjønnes [2] is

$$L(g) = g \sqrt{1 - \left(\frac{g}{2R_0}\right)^2}$$

Where  $R_0$  is the precession scan angle in reciprocal Angstroms. While this is an interesting idea unfortunately to date it has not been particularly successful as illustrated in Fig. 26.3.

## 26.5 1s Channeling Model

The concept of a channeling approach is to expand the electron wave in terms of local orbitals rather than plane waves, e.g. [53–56]. One can then approximate by using just the 1s states, which works well for HREM and STEM imaging [57].



**Fig. 26.3** Scatter plot of Lorentz-corrected data (y-axis) versus the true values (x-axis) for  $(\text{Ga}, \text{In})_2\text{SnO}_4$  with two different precession angles and three different thicknesses

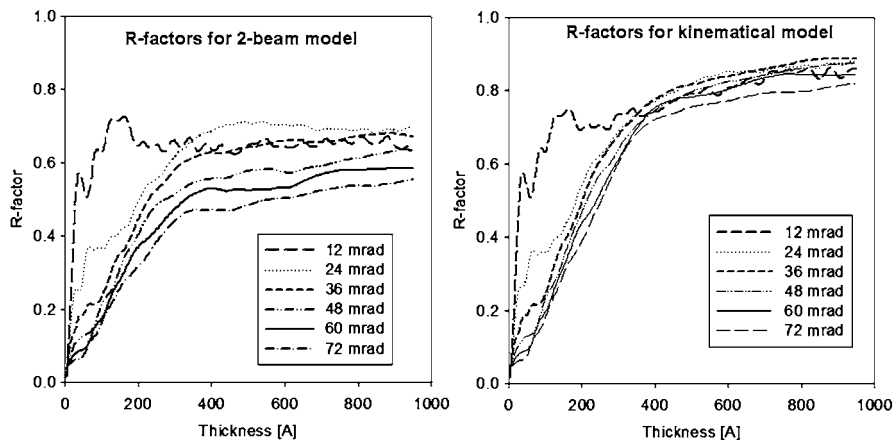
At least in order of simplicity, this model is an attractive approach. The result of the model is “atom-like” features and it has been shown that even though the results are dynamical, the deviations from kinematical are in fact statistical in character rather than being systematic [58, 59]. Since both direct methods and refinements are (in principle) stable against statistically random deviations, it is therefore true that in some cases on a zone axis these methods will work well. Alas, while there may be some relationship to what one finds in a PED pattern, to date this approach has not proved to be useful. (A 1s-model leads to scattering which is dominated by atomic strings which is similar to what PED yields so there may be some connection, but so far there is no proof beyond qualitative intuition.)

## 26.6 Two-Beam Model

The first model to account for at least some of the effects present is a two-beam model with a proper tracking of the range of integration. A specific form [26] is

$$C_{2beam}(g, t, \phi) = F_g^2 \left( \frac{1}{\xi_g^2} \int_0^{2\pi} \frac{\sin^2(\pi t s_{eff})}{(s_{eff})^2} d\theta \right)^{-1}$$

Where the effective excitation error  $s_{eff} = (s^2 + \xi_g^2)^{1/2}$  is used. This is better, but again not perfect and breaks down for a thickness much beyond 10 nm as illustrated in Fig. 26.4.



**Fig. 26.4** Comparison for  $(\text{Ga,In})_2\text{SnO}_4$  of the R1 for a two-beam model versus thicknesses (*left*) compared to a kinematical model (*right*). Unfortunately while there is some improvement, it is not enough

## 26.7 Full Multislice or Bloch Wave Methods

Good agreement between experimental and calculated intensities has been obtained using methods where all the dynamical diffraction effects (except fine details of inelastic scattering/adsorption) are taken into account. These are based upon either the multislice method [60–63], a fast numerical integration of the intensities, or Bloch Wave methods [64–66] where a matrix problem is solved. Assuming that the potential used is the same for the two methods, it is known that they give identical results provided that they have been properly coded.

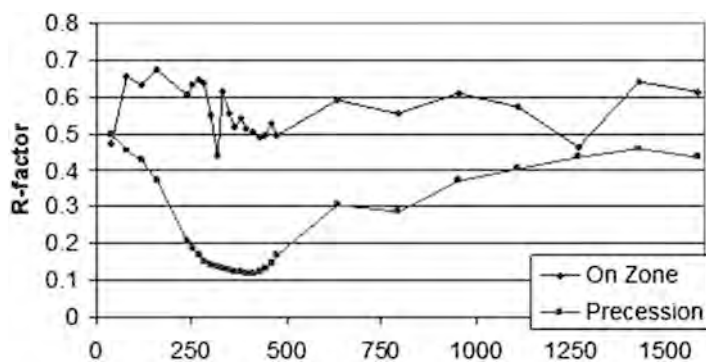
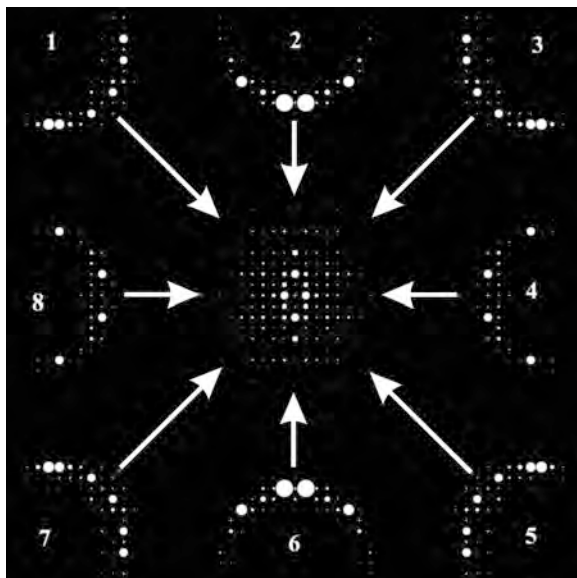
The approach [13], as illustrated in Fig. 26.5 is to consider all different incident beam directions and integrate the final intensity over these, for instance the set 1–8 below.

Without any additional refinement one can easily obtain an R1 of about 0.1, as illustrated below in Figs. 26.6 and 26.7.

## 26.8 Intensity Ordering

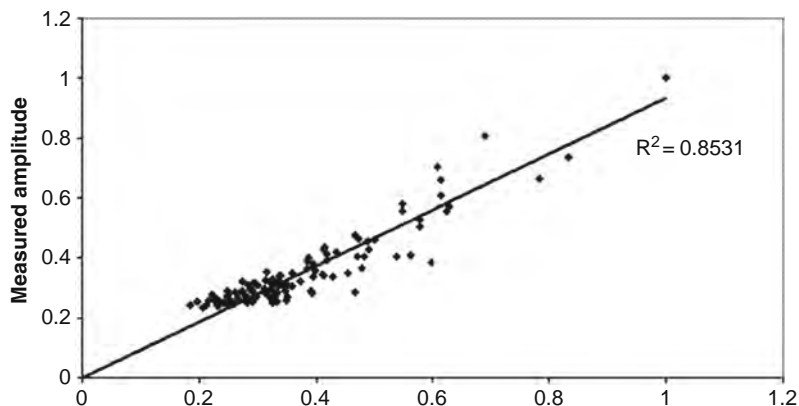
An explanation of why the methods work, which unfortunately slightly begs the question of the details of when they will fail, is intensity ordering [67]. Instead of the intensities being simply related to the structure factors as in kinematical or Blackman approaches, the hypothesis of this model is that reflections with large structure factors lead to large intensities in PED, those with small structure factors small intensities. By inspection this is largely true for the plots shown above which

**Fig. 26.5** Schematic of a dynamical simulation. For a range of different tilts a full calculation is performed and the results are summed. Specific results for eight illustrative tilts are shown; in general 512–1,024 different values are used



**Fig. 26.6** R1 as a function of thickness in Angstroms from a multislice calculation using experimental data for  $(\text{Ga,In})_2\text{SnO}_4$  both on-zone (*upper line*) and precessed (*lower line*). The minimum with precessed data is much clearer, and the R1 much lower

plot the kinematical structure factors versus the true values. This is a sufficient condition for direct methods to work, indeed in the early days of the technique with “by eye” measurement of intensities for x-ray diffraction from film, structures were solved by dividing the intensities into those which were strong, those which were of medium intensity and the weak ones. Classical direct methods only use the strong intensities, so provided that these are representative then  $\Sigma_2$  and similar relationships will be preserved.



**Fig. 26.7** Plot of measured amplitudes versus multislice calculations for the optimum thickness shown in Fig. 26.6

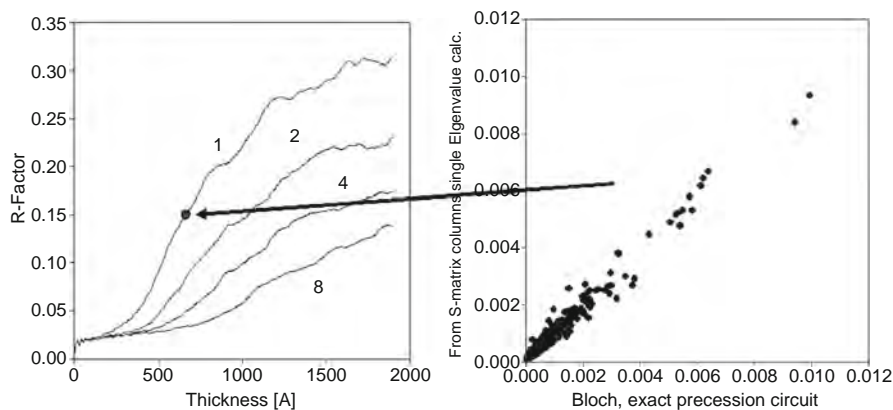
## 26.9 Summary

For certain PED has emerged as a powerful tool for solving structures. The intensities are much better behaved than those from zone-axis diffraction particularly if larger tilt angles are used. Unfortunately most simple models to date fail to explain fully the dynamical diffraction effects in enough detail so one has to do a full calculation.

Fortunately the PED intensities are not chaotic, but are ordered which is enough for direct methods to work and there is now extensive empirical evidence showing that this approach can be used to obtain an initial structure for later refinement either (or both) from powder x-ray data or by using a dynamical approach.

What remains as a problem is how to refine the structure, or perform structure completion – in most respects the later is a more significant issue as structure completion is in many respects why direct methods work. The large  $R1$  values with kinematical models are problematic. In principle one might be able to use a two-beam model as an improvement upon kinematical in a refinement as implied by an initial estimate [26] and one can use it to approximately invert a set of intensity data. This might be a viable refinement approach as it would be faster than a full dynamical method, and this is currently under investigation.

There are also other alternatives. For instance, some time ago it was suggested by Peng [68] that one could use a quasi-kinematical approach, an idea that may well be worth returning to. Alternatively there are ways to exploit the implicit periodicity in reciprocal space (Brillouin Zone folding) so rather than calculating 1,024 different tilts a much smaller number of Bloch wave calculations is needed, perhaps only 1 if chosen judiciously or at most 8 [47]. This could give a  $10^3$  improvement in speed and might make a Bloch wave refinement viable on a reasonable computer; full



**Fig. 26.8** Plot the R1 for  $(\text{Ga,In})_2\text{SnO}_4$  using a limited number of tilts exploiting Brillouin-Zone folding for different thicknesses relative to a full calculation (*left*) with a scatter plot of the intensities on the *right* for one Bloch-wave as *arrowed*

refinements will be unrealistically slow if all points are used. This is illustrated in Fig. 26.8 below which compares the results of an accurate Bloch wave calculation with 1,024 tilts to a much smaller set.

Despite these limitations, PED has moved from the early days when it was a curiosity to a mainstream tool for electron microscopists to use to determine structures where real-space imaging methods are problematic, for instance when there is beam damage or ambiguities in the interpretation of the images. Even with its current limitations the R1 values obtained are in most cases rather better than one can obtain with alternative approaches.

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